# Resonant production of secondary electrons generating a discrete energy structure in a magnetized electron beam system

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In a system of magnetized electron beams, multiplication of electrons can occur if the incident and return beams (the latter one is produced by secondary electrons emitted at the end of the incident beam) satisfy a resonance condition on the gyration phase. The beams propagate between two boundaries; one is an electron gun and the other is a Faraday cup detector. The resonance condition demands discrete energies (eigenvalues) of the Hamiltonian that generates a propagator of the beam. This model has been compared with Varma's formulation of discrete energies [R. K. Varma and A. M. Punithavelu, Mod. Phys. Lett. A **8**, 167 (1993)].

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## I. INTRODUCTION

Varma and Punithavelu [1] have shown that a surprisingly complex phenomenon can occur in a simple system of an electron beam propagating along a longitudinal magnetic field. Although the energy of the beam is in a totally classical-mechanical regime, the transport of electrons exhibits an unexpected discrete structure that resembles the quantum-mechanical conduction bands.

This problem was originally addressed by Varma [2]. He pointed out the similarity between the classical electron transport equation, after manipulation based on the recipe of the path-integral method [3], and the Schrödinger equation of matter waves. He predicted a band structure in a classical-mechanical electron beam in a magnetic field. However, an essential part of the model has not been explained with physical justification. A discrete spectrum stems from the Schrödinger-type formulation together with an appropriate ''boundary condition'' that must be imposed under a physical requirement. Formally, the Schrödinger operator may have a discrete spectrum when the boundary condition acts to trap the wave function in a bounded region. To explain the experiments, we need a periodic boundary condition for the wave function, or a ''propagator,'' that represents the transport of the electrons.

In this paper, we present a model of the ''resonance condition'' for electron multiplication due to secondary-electron emission at the boundary, which implies the desired boundary condition. The model gives a physical justification for the Schrödinger-type formulation and the corresponding discrete spectrum.

In Sec. II, we describe a concise summary of our experiment. There are some differences from the Varma-Punthavelu experiment. The model of secondary electrons generating a return beam is proposed in Sec. III. In Sec. IV, we present formulas describing the discrete energies based on the secondary electron model, and compare it with Varma's theory.

## II. ELECTRON BEAM EXPERIMENT

We begin with a review of the electron beam system that was first studied by Varma [2] and then explored experimentally [1]. We consider a very simple system consisting of an electron gun, a Faraday cup, and solenoidal coils that provide a longitudinal magnetic field. The electron gun consists of a filament, a cathode plate, and an anode plate. Each plate has a hole 2 mm in diameter. The Faraday cup (140 mm in diameter) consists of a collector plate, a repeller grid, and a shield grid. The transparency of each grid, is 49.7%. The background pressure is about $1\times10^{-7}$ Torr, and hence the scattering of electrons by residual gas particles is negligible. Figure 1 shows the electric potential field applied to the electrons in this system. The electrons move with a constant velocity between the anode plate and the shield grid. There is a minor difference from the Varma-Punithavelu experiment [1]. In their system, the Faraday cup does not have a shield grid, and hence the potential given at the repeller grid ex-

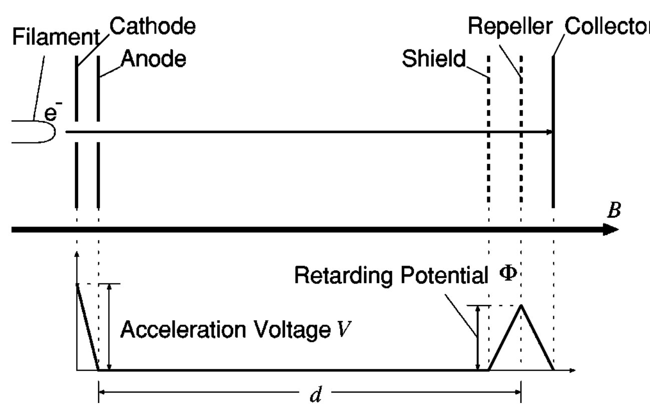


FIG. 1. Schematics of the experimental device and potential energy due to the external electric fields.

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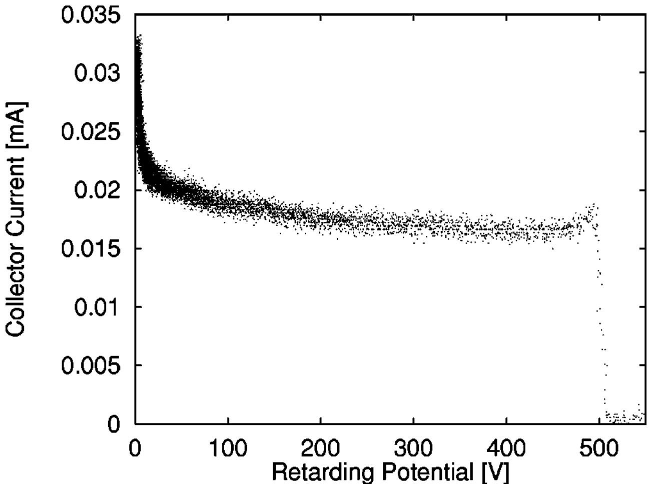


FIG. 2. Collector current as a function of retarding potential. $E=500$ eV, $d=24$ cm, and $B_{\parallel}=0$ G.

tends further. In the later discussion, however, this difference does not play an essential role.

When the magnetic field $B=0$ (see Fig. 2), for a repeller potential ($\Phi$) smaller than the acceleration voltage ($V$), all electrons reach the collector, while for $\Phi > V$, all electrons must be reflected at the repeller. A slight complication, however, appears because of the finite transparency of the electrode grids of the detector; the peak near $\Phi=0$ is due to secondary electrons emitted from the shield, which are only retarded by a small potential. When $B \neq 0$, on the other hand, very different results are obtained (see Fig. 3). We observed a number of peaks in the collector current. This result is similar to the previous experiment of Varma-Punithavelu [1], however there are some fundamental differences.

In our experiment, the wavy graphs such as in Fig. 3 (let us call this ''quantization'') are obtained only for discrete magnetic fields $B_{\ell}=\ell B_1$ ($\ell=1,2,3,\ldots$); $B_1$ is the minimum magnetic field for which the wavy structure appears. The value of $B_1$ depends on $d$ (the distance between the gun and the detector) and $V$ (acceleration voltage). In Fig. 4, we

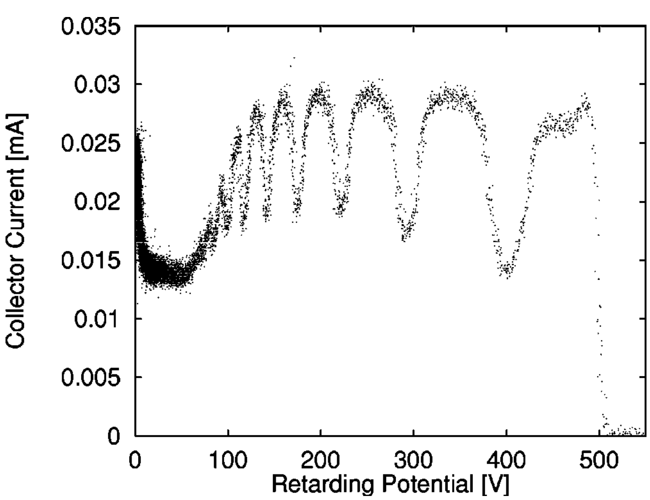


FIG. 3. Collector current as a function of retarding potential. $E=500$ eV, $d=24$ cm, and $B_{\parallel}=99.0$ G.

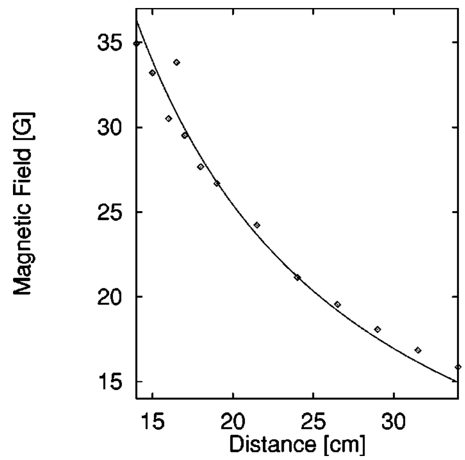


FIG. 4. Minimum magnetic field $B_1$ at which peaks appear as a function of distance $d$ ($V=600$ V). The solid line is the $B \propto d^{-1}$ curve.

plot $B_1$ as a function of $d$ ($V$ is fixed), and in Fig. 5, as a function of $V$ ($d$ is fixed). We find that the condition for the ''quantization'' is summarized as

$$\frac{\omega_c d}{v_e}=2\pi\ell \quad (\ell=1,2,3,\ldots), \tag{1}$$

where $\omega_c=eB/m_e$ is the cyclotron frequency and $v_e=(2eV/m_e)^{1/2}$ is the velocity of the electrons (nonrelativistic in the present energy range: $E \leqslant 1.5$ keV); see Table I. The left-hand side of Eq. (1) represents the gyration phase of the electron, and hence the condition (1) implies that the electrons must complete an integer number of gyrations (cyclotron motion) around the magnetic-field line during their travel between the gun and the Faraday cup. We can rewrite Eq. (1) in the form

$$eV=\frac{1}{2}m_e\left(\frac{\omega_c d}{2\pi}\right)^2\frac{1}{\ell^2} \quad (\ell=1,2,3,\ldots). \tag{2}$$

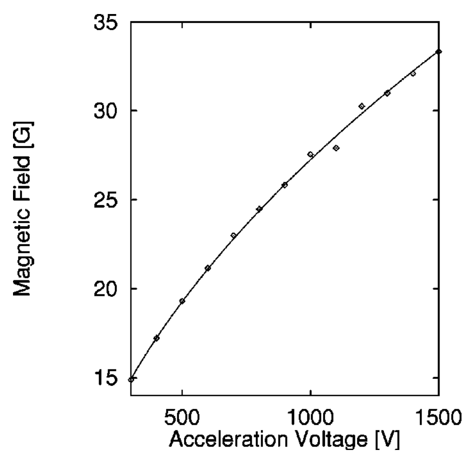


FIG. 5. Minimum magnetic field $B_1$ at which peaks appear as a function of acceleration voltage $V$ ($d=24$ cm). The solid line is the $B \propto V^{1/2}$ curve.

TABLE I. Comparison of experiment with relation (2) for magnetic fields at which peaks appear. $E=500$ eV and $d=24$ cm.

| $\ell$ | $B_\ell$[G] | |
|---|---|---|
| | Experiment | Relation (2) |
| 1 | 19.6 | 19.7 |
| 3 | 59.0 | 59.1 |
| 5 | 99.0 | 98.6 |
| 7 | 136 | 138 |

When quantization occurs, another discrete-number law applies to the retarding potential $\Phi$; the current peaks appear if

$$e\Phi=\frac{1}{2}m_e\left(\frac{\omega_c d}{2\pi}\right)^2\frac{1}{n^2}\quad (n=\ell,\ell+1,\ell+2,\ldots). \tag{3}$$

In Fig. 3, the ''quantum numbers'' $n$ of the peaks are 5,6,7, . . . . However, the difference between the measurement and the relation (3) becomes larger (up to 10%) for smaller voltages ($\Phi<100$ V). We note that the peaks in Varma's experiment [1] satisfy the relation

$$e\Phi=\frac{1}{2}m_e\left(\frac{3\omega_c d}{2\pi}\right)^2\frac{1}{n^2} \tag{4}$$

rather than Eq. (3). The equivalence of Eqs. (3) and (2) suggests that the ''quantization'' is related to the gyration phase of electrons. In the following section, we will explain this phenomenon by a simple model.

## III. RETURN BEAM GENERATED BY SECONDARY ELECTRONS

Secondary electrons play an essential role in the present model. When beam electrons bombard the repeller grid of the Faraday cup, secondary electrons are produced. These electrons are accelerated by the retarding potential $\Phi$ and go back to the gun. Neglecting the initial energy of secondary electrons, we obtain a ''return beam'' of energy $e\Phi$. Each electron gyrates with Larmor radius $r_L=m_e v_\perp/eB$ , where $v_\perp$ is the velocity perpendicular to the magnetic field. For incident beam electrons, $v_\perp$ is generated by the emittance of the beam generated at the cathode. But for return beam electrons, $v_\perp$ is determined by the initial velocity of secondary electrons emitted in a random direction. The return beam focuses at every $2\pi$ of the gyration phase, if the return beam is generated at a concentrated place on the grid, i.e., the incident beam focuses on the grid [Fig. 6(a)]. This condition on the incident beam is described by Eq. (2). Under this condition, the return beam can focus on the front of the electron gun when the relation (3) holds [Fig. 6(b)]. Then, almost all of the return beam electrons can enter the hole of the electron gun and be reflected by the acceleration potential [Fig. 6(c)]. The Faraday cup detects the total number of electrons that pass through the two grids. We thus see that the electrons are multiplied by the incident and return beams when the ''resonance conditions'' (2) and (3) on the gyration phase are satisfied.

Using this model, we made a computer simulation of the collector current with changing retarding potential. Here, we assume that the probability that electrons pass through the grid is $\frac{1}{2}$ and the energy of secondary electrons $E_0$ is 5 eV. The perpendicular energy of incident beam electrons $E_\perp$ is set to 3 eV. We assume that all secondary electrons emitted from the repeller are accelerated toward the gun, and all of those that enter the hole of the anode are again reflected and have enough energy to cross both grids because $e\Phi+E_0>e\Phi$. The secondary electron yield $\delta$ is set to 3 in the simulation. The result of the simulation shown in Fig. 7 reproduces the quantization of the collector current. However, the simulation with a reasonable secondary electron yield gives only small and narrow peaks, while in the experiment more drastic changes were observed. This may be resolved by studying the secondary emission process more carefully.

## IV. DISCRETE STRUCTURE IN THE ENERGY OF ELECTRONS

In this section, we introduce a propagator that represents the transport of the electrons, which is the eigenfunction of a formal Schrödinger operator, and then we incorporate the ''resonance condition'' of Sec. III into the model as a ''boundary condition'' for the propagator. We will show that the eigenvalues of the Hamiltonian give the energy at which electron multiplication occurs. The general idea of describing discrete energies by an eigenvalue problem is similar to Varma's original theory [2]. However, the physical argument of introducing the wave function (propagator) and the result of the eigenvalue problem is different.

We formulate the eigenvalue problem by involving the Hamilton-Jacobi equation for the action integral. We con-

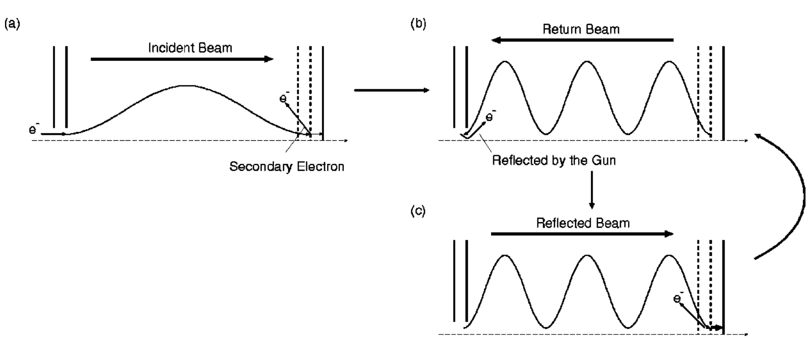


FIG. 6. Schematics of electron orbits of (a) an incident beam, (b) a return beam, and (c) a reflected return beam.

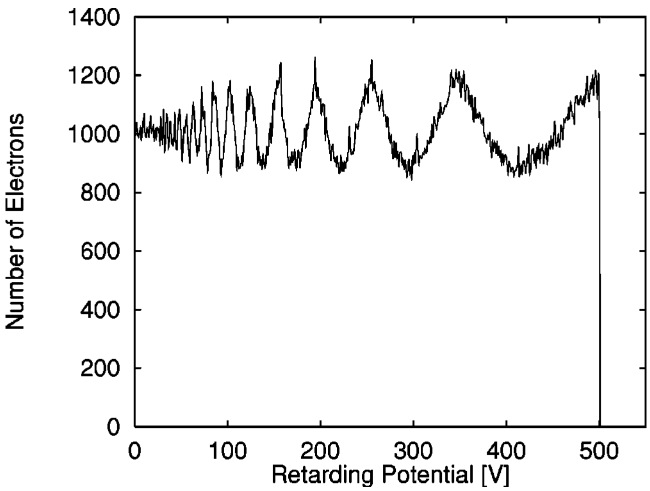


FIG. 7. Collector current as a function of retarding potential (computer simulation). $E=500$ eV, $d=24$ cm, and $B_{\parallel}=99.0$ G. Total number of electrons is 3000.

sider a unitary transform (propagator) $\Psi$ associated with the action integral. $\Psi$ is different from the wave function $\varphi$ such that $f=\varphi\varphi^*$ satisfies the classical Liouville equation (Varma's formulation [2]). The classical Hamiltonian of a charged particle in a homogeneous magnetic field is

$$H=\frac{1}{2m}p^2+\mu\omega_c, \tag{5}$$

where $p=mv$ is the momentum in the direction parallel to the magnetic field, and $\mu=\frac{1}{2}mv_\perp^2/\omega_c$ is the magnetic moment (adiabatic invariant). The $\mu\omega_c$ can be regarded as a potential energy. We define

$$\Psi=e^{iS/\mu}, \tag{6}$$

where $S=\int L\,dt$ is an action integral for the adiabatic Lagrangian $L=(1/2m)p^2-\mu\omega_c$ [2]. Since $v$ and $\omega_c$ are constant, Eq. (6) becomes

$$\Psi(t)=\exp\left[i\left(\frac{1}{\mu}\frac{p^2}{2m}t-\omega_c t\right)\right]. \tag{7}$$

For $S$ evaluated through the actual motion, we can rewrite Eq. (7), using $t=z/v$, as

$$\Psi(z,t)=\exp\left[i\left(\frac{1}{\mu}\frac{p^2}{2m}t-\frac{\omega_c}{v}z\right)\right]. \tag{8}$$

In Eq. (8), the resonance condition for the gyration phase (see Sec. III) translates into the periodic boundary condition on $\Psi(z,t)$ over the distance $d$ between the gun and the detector, i.e.,

$$\Psi(0,t)=\Psi(d,t), \tag{9}$$

which demands

$$\frac{\omega_c d}{v}=2\pi\ell \quad (\ell=1,2,3,\ldots). \tag{10}$$

The time-dependent part of $\Psi$ in the form of Eq. (8) gives the eigenvalues of the parallel kinetic energy:

$$-i\mu\frac{\partial\Psi}{\partial t}=E\Psi=\frac{p^2}{2m}\Psi. \tag{11}$$

Substituting Eq. (10) into Eq. (11), we can get the discrete energy eigenvalues

$$E=\frac{1}{2}m\left(\frac{\omega_c d}{2\pi}\right)^2\frac{1}{\ell^2} \quad (\ell=1,2,3,\ldots). \tag{12}$$

These eigenvalues correspond to the experimental results of quantization (2) and (3). The present classical mechanical system is described by the propagator with finite $\mu$ $(=\frac{1}{2}mv_\perp^2/\omega_c)$, which parallels $\hbar$. The Larmor radius $r_L$ $(=v_\perp/\omega_c)$ is comparable to the size of the hole on the anode, and hence the finiteness of $\mu$ is essential to determine the discrete beam energy.

In Varma's formulation [2], the discrete energies (12) are derived by taking the variation of $\mu$ for the periodic boundary condition, which includes unknown $\mu$. These energies, however, conflict the eigenvalues of the Hamiltonian operator given in his theory. The present theory does not need such a treatment for the boundary condition and does not include contradictions in estimates of energies.

## V. DISCUSSION

The discrete energy structure in the electron beam system is explained by the resonance condition that gives rise to electron multiplication through secondary electron emission at the electrode. The resonance condition on the gyration phase can be formulated as a periodic boundary condition on the formal propagator, and the corresponding eigenvalue of the energy gives the discrete energy at which electron multiplication occurs. There are several differences, in both experimental observation and theoretical formulation, from Varma's original work. The present theory and experiment fill gaps and correct flaws left after this pioneering work. However, there are still some unresolved problems that require careful study: (i) The difference between the measurement (Fig. 3) and the relation (3) becomes larger (up to 10%) for smaller voltages ($\Phi<100$ V); (ii) the simulation (Fig. 7), with a reasonable secondary electron yield, gives only small and narrow peaks, while in the experiment more drastic changes were observed. These questions may be resolved by studying the secondary emission process more carefully.

## ACKNOWLEDGMENT

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